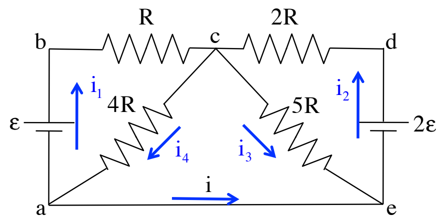
Problem 28.27

Assuming $R = 1\ k\Omega$ and $\varepsilon = 250$ volts, what is the direction and magnitude of current between Points "a" and "e?"

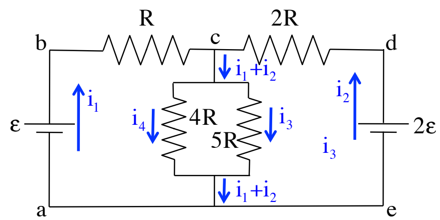
I've assumed current directions and labeled them on the sketch.



Looking at what we have, the Node equation for node "a" is:

$$i_4 = i_1 + i \qquad \text{(equation A)}$$

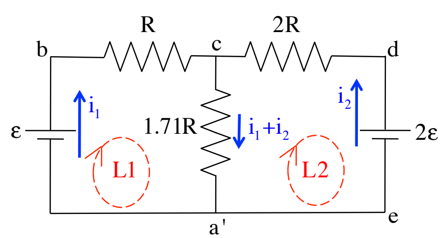
Apparently, we can determine "i" if we can determine the other two currents. It isn't always useful, but sometimes it makes sense to re-draw a circuit putting it into a geometry that is more familiar. That is what I've done to the right. (If you can't see that the two circuits are essentially the same, talk to someone about it!)



The equivalent resistance of the internal parallel combination is:

$$R_{equ} = \left(\frac{1}{4R} + \frac{1}{5R}\right)^{-1}$$
$$= 1.71R$$

With that, the circuit can be drawn as shown to the right.



With both start at "a' " and traverse as shown, the Loop Equations yield:

$$\text{L1:} \qquad \varepsilon - i_1R - (i_1 + i_2)(1.71R) = 0$$
$$\text{L2:} \qquad (i_1 + i_2)(1.71R) + i_2(2R) - 2\varepsilon = 0$$

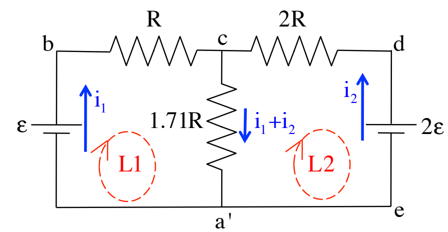
Solving these simultaneously yields:

$$i_1 = 10\text{x}10^{-3}\ A$$
$$i_2 = 1.3\text{x}10^{-1}\ A$$

(If I have time, I'll come back and actually do the solving at the end of the problem . . . for those of you who don't like algebra or matrix manipulation):

The current through the 1.71R ohm resistor is:

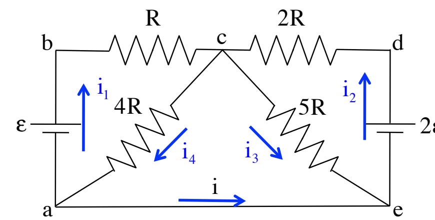
$$i_1 + i_2 = (10\text{x}10^{-3}\ A) + (130\text{x}10^{-3}\ A)$$
$$= 140\text{x}10^{-3}\ A$$



So the voltage across that central resistor is:

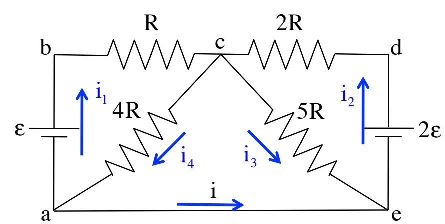
$$V_{c-a'} = (i_1 + i_2)(1.71R)$$
$$= (140\text{x}10^{-3}\ A)(1.71(1000\ \Omega))$$
$$= 240\ V$$

But because the real resistors making up this 1.71R equivalent resistance are in parallel, that voltage must also be the voltage across each of those elements . . . which brings us back to our original circuit.



Knowing the voltage across the 4R resistor is 240 volts, we can write:

$$V_{c-a} = i_4(4R)$$
$$\Rightarrow \quad i_4 = \frac{240\ V}{4(1000\ \Omega)}$$
$$= .06\ A$$



And with that information, we can write out that original node equation as:

$$i_4 = i_1 + i$$
$$\Rightarrow \quad i = i_4 - i_1$$
$$= (.06\ A) - (.01\ A)$$
$$= .05\ A$$

Finally, the calculated value for "i" was positive, so we know the originally assumed direction was good. Current flows from "a" to "e."